New Einstein Gravitation*

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Abstract: Taking into account the experimental accuracy of the body motions in the solar system, we propose a General Relativity Theory which is new but nevertheless completely metric.

Space-time or Space and Time

Let us first recall some experimental results. The atomic clocks are so precise that we can check the relativistic effects. Each clock follows and measures its own proper time as it is defined in general relativity(1). The UAI (Union Atomic Inernational) recommends to choose $(x^0 = ct, x^1, x^2, x^3)$ and

$$ds^2 = -c^2(d\tau)^2 = -(1 - 2U/c^2)(dx^0)^2 + (1 + 2U/c^2)(d\vec{x})^2$$

where c is the light velocity, τ the proper time $U > 0$ is the Newton gravitational potential of all the masses...With such formula it is possible to correct each individual clock by measuring its apparent acceleration and so to obtain a local synchronization which permits to define the time coordinate t and a 3-dimensional hypersurface. An unexpected result is that all objects are allways confined in this subspace. But this is impossible since in Minkowski space such subspace depend of the arbitrairy choice of a reference galilean frame (geocentric or barycentric). So we have to change our window and to admit that we are in a 3-dimensional space with a flow of time. Each observer build its time and its own Minkowski 4-space (x^0, x^1, x^2, x^3) from the same time and the same space (2).

Let us now reconsider general relativity to see the impact of such result. We will adopt the requirement claiming that everythings in general relativity must be derived from a given metric :

$$(ds)^2 = \eta_{\mu\nu}\omega^\mu\omega^\nu = -c^2(\omega^0)^2 + (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2$$

but we will utilise all resource of the Cartan formalism in such metric geometry to construct the theory.

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Cartan Formalism in Metric Spaces

Cartan connexion

$$d\vec{m} = \omega^{\mu}\vec{e_{\mu}} \qquad d\vec{e_{\mu}} = {\omega_{\mu}}^{\nu}\vec{e_{\nu}}$$

Galilean connexion

$$\omega^0 = dt \qquad {\omega_i}^j = -{\omega_j}^i \qquad {\omega_i}^0 = 0$$

Lorentz connexion

$$ {\omega_i}^j = -{\omega_j}^i \qquad {\omega_i}^0 = c^{-2}{\omega_0}^i$$

Torsion and curvature

$$\Omega^{\mu} = d\omega^{\mu} - \omega^{\rho} \wedge {\omega_{\rho}}^{\mu} \qquad {\Omega_{\mu}}^{\nu} = d{\omega_{\mu}}^{\nu} - {\omega_{\mu}}^{\rho} \wedge {\omega_{\rho}}^{\nu}$$

From the Hilbert elementary action

$$\sum_{(\mu\nu\rho\lambda)} \omega_{\mu} \wedge \omega_{\nu} \wedge \Omega_{\rho\lambda}$$

Cartan derives the energy-momentum torque

$$G = \sum_{(\mu\nu\rho\lambda)} \Big(\vec{e}_{\mu}(\omega_{\nu} \wedge \Omega_{\rho\lambda} + \omega_{\rho} \wedge \Omega_{\lambda\nu} + \omega_{\lambda} \wedge \Omega_{\nu\rho}) - \vec{e}_{\mu} \wedge \vec{e}_{\nu}(\omega_{\rho} \wedge \Omega_{\lambda} - \omega_{\lambda} \wedge \Omega_{\rho}) \Big)$$

It is formed of a vector with components

$$G^{\mu} = \omega_{\nu} \wedge \Omega_{\rho\lambda} + \omega_{\rho} \wedge \Omega_{\lambda\nu} + \omega_{\lambda} \wedge \Omega_{\nu\rho}$$

and a bivector with components

$$G^{\mu\nu} = \omega_{\rho} \wedge \Omega_{\lambda} - \omega_{\lambda} \wedge \Omega_{\rho}$$

The exterior derivative of G is not zero but a vector with components

$$F^{\mu} = \Omega_{\nu} \wedge \Omega_{\rho\lambda} + \Omega_{\rho} \wedge \Omega_{\lambda\nu} + \Omega_{\lambda} \wedge \Omega_{\nu\rho}$$

In a Riemann connexion which has by definition no torsion

$$\Omega_{\mu} = 0 \qquad G^{\mu\nu} = 0 \quad \text{and} \quad F^{\mu} = 0$$

and G^{μ} turn out to be just the 3-form build up from the famous conserved Einstein tensor $R^{\mu\nu} - 1/2g^{\mu\nu}R$.

In the following we will utilise also two well-known results

The Weyl Theorem:

If $\Omega^\mu - d\omega^\mu$ is given by $\lambda_\rho{}^\mu{}_\nu \omega^\rho \wedge \omega^\nu$ then

$$\omega_{\mu\nu} = \gamma_{\mu\nu\rho}\omega^\rho \quad \text{where} \quad \gamma_{\mu\nu\rho} = \lambda_{\mu\nu\rho} - \lambda_{\nu\rho\mu} + \lambda_{\rho\mu\nu}$$

The Cartan Lemma:

If all $\vec{e}_\mu \wedge \vec{e}_\nu(\omega_\rho \wedge X_\delta - \omega_\delta \wedge X_\rho) = 0$ where X_δ are 2-forms then

$$\text{all} \quad X_\delta = 0$$

The new Theory

Since each atomic clock indicates its own proper time we must impose

$$\Omega^0 = d\omega^0 - \omega^i \wedge \omega_i{}^0 = 0$$

and via the synchronization we obtain

$$\omega^0 = f(x)dt$$

Let us now verify that according to Weyl Theorem we can choose

$$\omega_0{}^i = -Y^i\,\omega^0 \qquad \text{and} \qquad \omega_i{}^j = 0\,.$$

Proof

By definition the coefficients Y^i are such that

$$\Omega^0 - d\omega^0 = -d\omega^0 = -df \wedge dt = -c^{-2}\,Y_i\,\omega^i \wedge \omega^0\,.$$

Then we have

$$\lambda_i{}^0{}_0 = -1/2c^{-2}\,Y_i \quad \text{and} \quad \lambda_i{}^0{}_j = 0$$

and so by Weyl Theorem

$$\gamma_{0i0} = \lambda_{0i0} - \lambda_{i00} + \lambda_{00i} = -Y_i$$

and

$$\gamma_{0ij} = \lambda_{0ij} - \lambda_{ij0} + \lambda_{j0i} = 0\,.$$

Thus we find

$$\omega_{0i} = \gamma_{0i0}\,\omega^0 + \gamma_{oij}\,\omega^j = -Y_i\,\omega^0$$

and

$$\omega_{ij} = \gamma_{ij0}\,\omega^0 + \gamma_{ijk}\,\omega^k = 0\,.$$

From these we derive the following results

$$\Omega^i = d\omega^i \qquad \Omega_i{}^j = 0 \quad \text{and} \quad \Omega_0{}^i = d\omega_0{}^i$$

The space has no curvatur but it is not flat since its torsion is not zero.

Let us now recall the Newton-Cartan Theory :

The torsion $\Omega^{\mu} = 0$, the space is flat, all ${\omega_i}^j$ can be choosed to be zero and the last one ${\omega_0}^i$ is the Newton gravitational acceleration $-F^i\,\omega^0$. In such connexion the physical trajectories are just straight lines.

In our relativistic theory we have ${\omega_0}^i = -Y^i\,\omega^0$, the Einstein gravitational acceleration but, here of course, ${\omega_i}^0$ is not zero since it is a Lorentz connexion.

The straight lines are not exactly the geodesics but this makes no difference for the prediction of the Mercury perihelion shift (4).

On the other hand the Cartan torque reduces to a space vector

$$G^i = \omega_j \wedge \Omega_{k0} + \omega_k \wedge \Omega_{0j}$$

and a bivector

$$G^{0i} = \omega_j \wedge \Omega_k - \omega_k \wedge \Omega_j$$

According to the Cartan Lemma, $G^i = 0$ impose $\Omega_{0k} = d\omega_{0k} = 0$ and the gravitational acceleration is uniform. More $G^{0i} = 0$ impose $\Omega_k = 0$, the space is flat. All these are in agreement with the hope of Einstein himself before he knews Schwarzschild results.

Références

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Further Clarification of "New Einstein Gravitation"

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I wish to explain further here the ideas of Cartan and my paper "New Einstein Gravitation".(1) It is not a question of constructing a different theory, but another interpretation, taking into account the fault hinted at by Einstein that the flow of time is an illusion and therefore that particles follow their trajectories according only to their proper times. When Cartan gives in the tangent space at each point a frame adapted to the group considered, he does not specify the system of coordinates, and still less a frame naturally defined by the coordinates; there are therefore no *vierbeins*, the objects with two indices. The tangent space is in fact a notion intrinsic to the manifold. The form ω puts into correspondence to a vector f a number or an element of an algebra. The exterior product is defined in general as

$$\omega_1 \wedge \omega_2(f_1, f_2) = \omega_1(f_1)\omega_2(f_2) - \omega_1(f_2)\omega_2(f_1)$$

Therefore one permutes the vectors and not the values taken by the forms. For the particles following trajectories which are timelike, one can make the assumption with Einstein that these are geodesics and so one does not have the interpretation of spacelike curves in R^4. One requires a connection, but why to choose that of Lévi-Civita? On the other hand, Einstein is mistaken; the particles remain confined in an R^3 contained in R^4, but then which one? The answer is simple. It is the one which has been take at the beginning to construct the R^4, and therefore in a change of the Galilean frame the R^3 is the same and it is the R^4 that changes. One must therefore define a connection for this R^3 which is now non-Euclidean (this is

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the inspired idea of Einstein). This is why I have chosen the connection for which $\omega_{ij} = \omega_{ji} = 0$ for such R^3, which then necessarily has torsion (see my paper Ref. 1). It is not correct to deduce that an object which moves freely does not turn, but there is not in addition a rotation due to the Lévi-Civita connection, a very little term (see the reference to Garid-i in my paper Ref. 1). The predictions remain almost the same, and the theory is preserved.

REFERENCE

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