Quaternions

The quaternion are the elements of $\mathcal{Q}$, a four-dimensional vector space on the field of reals with basis elements e, j, k, l and the rules of multiplication

$$ee = e \qquad jj = kk = ll = -e$$

$$ej = je = j \quad ek = ke = k \quad el = le = l$$

$$jk = l \qquad kj = -l$$

$$kl = j \qquad lk = -j$$

$$lj = k \qquad jl = -k$$

Such structure seems very promising for the description of space-time, since e can be compared with $\vec{e}_0$ and j, k, l with respectively $\vec{e}_j\,, \vec{e}_k\,, \vec{e}_l\,$.

But a difficulty appears with the product, since $\vec{e}_j \wedge \vec{e}_k$ even in R^3 is not a vector; it is an axial vector and must be noted $\breve{e}_l$ as for the magnetic field $\breve{B}$. Moreover you can not continue the correspondence since $\breve{e}_l \wedge \breve{e}_k$ has no meaning. In fact such an expression never appears in Maxwell theory.

Given at the IARD 2004 Conference, Saas-Fee Switzerland